A Software GPS Receiver for Weak Signals

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Abstract — The paper reports the approach and results of a stand alone software global positioning system (GPS) receiver. The receiver is specially designed to receive weak signals without aids from other information sources. The approaches for the acquisition, first navigation phase detection, and tracking are discussed. The different kinds of thresholds with different signal-to-noise ratio (S/N) are developed. Under the best operating condition a signal can be 15 dB below the normal noise level and still be processed.

I. INTRODUCTION

Under normal conditions GPS signal strength is about -130 dBm in a bandwidth of about 2 MHz where the thermal noise at room temperature is about –111 dBm. Thus, the equivalent S/N is about -19 dB (-130+111). Another common way to represent the S/N is C/N_0, which is referenced to 1 Hz bandwidth. Since the thermal noise per Hz at room temperature is –174 dBm, the C/N_0 = 44 dB (-130+174). Under this signal strength, the signal can be easily processed.

When a GPS is operating in an urban area or inside a building, buildings will block and attenuate the signal. If a receiver is under jamming the S/N also decreases. It is always desirable to improve the sensitivity of a GPS receiver. A software receiver is specially designed to process weak GPS signals. In order to have accurate S/N, simulated signals are used for the study.

In receiving conventional GPS signals 1 ms of data is usually adequate for all the processing. This means one can acquire the signal with 1 ms of data and track the signal with data 1 ms long. In order to process weak GPS signals the acquisition and tracking become much more difficult and the data needed for processing is much longer. In order to track long data, the first navigation phase transition should be detected. Thus, there are three steps to process weak GPS signals: acquisition, 1^{st} phase detection and tracking.

The hardware used to collect GPS signals down converts the input from 1575.42 MHz to 21.25 MHz and digitizes it at 5 MHz [1]. With this arrangement, 1 ms of data contains 5000 points. The simulated data is generated the same way. In order to study the sensitivity of the receiver, a simulated Gaussian noise with variance equal to 1 is added to the signal. The signal is scaled down based on the specified S/N. All the data generated in this report uses this assumption. Since the GPS signals received in the real world are noise dominated (S/N < -19 dB), the results generated in this report can be applied to the real signal. If the standard deviation of the received signal is measured the results in this paper can be factored by the standard deviation.

II. ACQUISITION

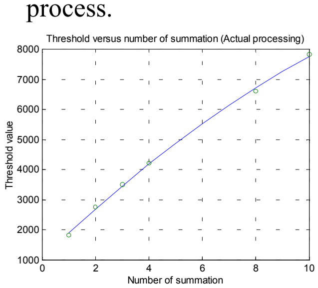
Acquisition is the most important step to a GPS receiver because one must detect the signal. The acquisition generates two important parameters: the carrier frequency and the initial phase of the C/A code. In general, acquisition performed on long data will increase the receiver sensitivity. An efficient way to perform acquisition is through circular correlation in the frequency domain [2] [3]. The number of operations is proportional to $N \times \log_2 N$ where N is the number of data. When N increases the number of operations will increase tremendously. It limits the total number of data points used in the acquisition. A new circular correlation method through partition [4] can greatly decrease the number of operations and make long data acquisition possible.

The potential occurrence of a navigation data phase transition in every 20 ms also limits the data length for acquisition. Any section of data can contain a phase transition. However, if two consecutive10 ms of data are selected for acquisition, it is guaranteed that one set of the 10 ms data is phase transition free. If the 10 ms data sets are assigned with a consecutive number, the phase transition can happen in either odd or even number sets. The acquisition method selected is to continue non-coherent summation of 10 ms coherent integrated data until the signal is detected or 10 summations is reached. The acquisition results obtained from the odd number sets are summed and the results of even number are also summed. The corresponding thresholds after every summation are pre calculated. After each summation, the result is compared against the proper threshold. If the result passes the threshold the acquisition is completed.

The results of the acquisition are the initial phase of the C/A code and the coarse carrier frequency.

A. Threshold determination

In order to determine the sensitivity of a receiver the false alarm and the probability of detection must be specified. The detection threshold is obtained by the Monte Carlo approach. A simulated noise with variance equal to 1 is input to the pre-defined acquisition process. The simulation is run 100 times with noise sequence regenerated each time. The maximum of the 100 peaks of the 100 runs is selected. The whole process is performed 10 times and results in 10 data points. The threshold is defined by the average of these 10 data points. Different thresholds are developed for different number of non-coherent summations. The result is shown in Figure 1a. This threshold can be scaled to any noise level by its standard deviation because the acquisition is a linear process.



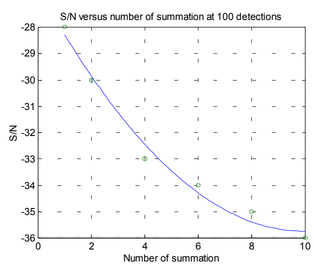


Fig. 1a Threshold versus number of summations
Fig. 1b S/N versus number of summation for 100 detection

B. Probability of detection

The best operational conditions are defined as when the input frequency is coincident with one of the frequency bin, and the digitization of the data starts at zero phase of the C/A code. This condition is used for testing the probability of detection against the threshold. The signal with the same S/N is input to the acquisition process and run 100 times. For each run, different noise is generated and the number of detection is recorded. Figure 1b shows the minimum S/N required for detecting the signal 100 times in 100 runs versus number of summations. This curve is similar to the non-coherent integration in processing radar pulses [5]. Sensitivity increases greatly in the first several summations, and the improvement saturated afterward. From this curve it appears that the optimal number of summation is about 6 and the improvement is close to –34 dB.

The worst operating conditions is when the frequency of the input signal is in between bins and the digitization of the signal is100ns off the code phase. The sensitivity can lose 3.9 dB due to the frequency offset and 0.92 dB from the code offset. Therefore, the overall degradation should be about 5 dB. From limited experiments, when the signal frequency is between two bins, these two adjacent frequency bins contain the energy of the signal. If they are summed together it can improve the S/N by about 2 dB. In other words, under the worst signal condition the sensitivity will decrease 3 dB rather than 5 dB. For example, with 6 summations the worst case sensitivity should be –31 dB, which is slightly better than the –29 dB anticipated.

III. Fine Frequency Calculation

Using 10 ms of data to perform acquisition by coherent integration, the frequency resolution is 100 Hz. Thus, the frequency accuracy should be ±50 Hz. If the signal is strong, this value is adequate to find the phase transition. For weak signals, the closer the measured frequency is to the true frequency, the more accurate the correlation peak will be. The correlation is used to determine the phase transition.

The approach to measure fine frequency is through the squaring of the input signal. After the initial phase of the C/A code is found in the acquisition, the C/A code can be stripped off from the input signal. But the phase transition caused by the navigation data still exists. It is well known that when a bi-phase-coded signal is squared, the phase transition can be eliminated and the output frequency is twice the input frequency. However, this method significantly decreases the signal to noise ratio, especially when the signal is below noise. To overcome this problem, before squaring the signal, the input frequency is down converted to a lower frequency of less than 1 kHz, such as 250 Hz. Each millisecond of data is averaged into one data point. The noise is greatly reduced in this averaging process. Thus, the equivalent sampling rate becomes 1KHz. After averaging, the signal is squared and over 200 ms of data (over 200 points) is used to perform a FFT (Fast Fourier Transform). The frequency resolution is less than 5 Hz and the total frequency coverage is 1 kHz. Since the frequency is doubled, the frequency accuracy can be less than ±1.25 Hz. Hundreds of simulations using data with a known frequency are run to determine the optimal data length for the software receiver. The results are shown in Table1. The criterion for a successful run is the measured frequency must be within ±2 Hz of the known frequency. From these results it appears that 256 ms of data is adequate for this software receiver and is also a good number to use with the FFT algorithm. The 256 data points are then down converted to 0±2Hz by the measured fine frequency. These 256 down converted data will be used to locate the first navigation phase transition in the following section.

Table 1 Number of successful frequency readings in 100 runs

S/N Data length (ms)	-33dB	-34dB	-35dB	-36dB
200	100	97	89	73
240	100	100	94	80
256	100	100	98	93
300	100	100	100	100

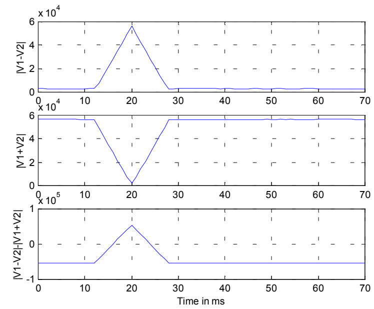


Fig 2 Comparison of $| V_1 - V_2 |$, $| V_1 + V_2 |$, $| V_1 - V_2 | - | V_1 + V_2 |$

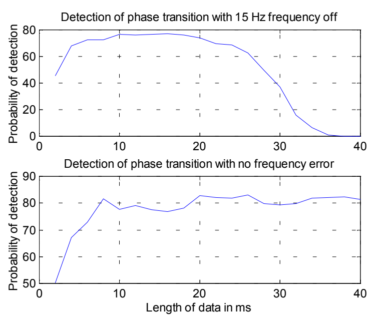


Fig 3a Probability of detection of phase transition versus data length (reference is15Hz off)
Fig 3b Probability of detection of phase transition versus data length (reference has no frequency error)

IV. First phase transition determination

To determine if there is a phase transition at time t_0 in 256 data points from the last section, let the complex number V_1 be the complex summation of 8 data before t_0, V_2 the complex summation of 8 data after t_0, and $V_t = |V_1 - V_2| - |V_1 + V_2|$. Figure 2 shows the results of a phase transition at time 20 ms. Hundreds of simulations have been run to compare the performance of V_t, $|V_1 - V_2|$ and, $|V_1 + V_2|$ as a discriminator for the phase transition. V_t becomes an obvious winner after these simulation runs. The reason why 8 data points are used for summation is based on the experimental results. The probability of detection versus the length of data for summation are shown in figure 3a and 3b,. In Figure 3a, the frequency error is 15 Hz and in Figure 3b there is no frequency error. These are the combined results from S/N of –28 to –36 dB. These results show that when there is frequency error, the long integration has adverse effects. When there is no frequency error the long integration does not have significant advantage over 8 ms data points.

Apply V_t on the 256 data points from previous section and pick the peak of the results. If the peak is larger than the threshold 270, which is based on 1% false detection criterion, a phase transition is detected at that location. The index of the location is used as reference to search every 20 ms for the phase transitions in these 256 data points. The observation window used in searching the phase transition is 9 points, ±4 point around the search point. If any point in this window passes the threshold 150, a phase transition is detected. The threshold is obtained from simulation based on the criterion of the equal opportunity of false detection and missed detection. The simulations with different S/N are run 10×100 times. The average number of failures in every 100 runs is 2.0 at S/N = -36dB, and is 0.4 at S/N=-35dB. In the weak signal environment, the detection of phase transition only tells that there is a transition in this neighborhood. In order to locate where the transition really occurs, the results of several observation windows with transition detected have to be added together. Table 2 shows the averaged number of location estimation failures in term of the number of windows summation and S/N. If the desired number of phase transition is not able to be detected in these 256 data points, more data from the in-coming signal is needed until the number of transitions required are detected.

Table 2 Averaged Number of failure to detect the location of phase transition in 100 runs (Averaged over 10 100-runs)

S/N Number of transition	-31dB	-32dB	-33dB	-34dB	-35dB	-36dB
5	0.4	0.8	1.6	3.0	5.3	7.8
6		0.2	0.9	2	3.2	5.5
7		0.2	0.3	1.2	2.0	4.1
8			0.2	0.6	1.5	2.35
9				0.6	0.8	1.7
10				0.2	0.8	1.4

V. Tracking

The purpose of the tracking is providing three parameters: the fine carrier frequency update, the navigation data phase transition and the position of the correlation peak of the C/A code. The fine frequency update is through phase measurement. Forty milliseconds

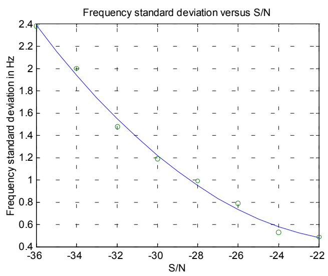


Fig. 4 Frequency error verse S/N

Table 3 The root mean square error in meters averaged in 10 runs

S/N Data integration	-32dB	-33dB	-34dB	-35dB	-36dB
20 ms	16m	17m	19m	32m	36m
40 ms	10m	12m	15m	17m	18m
80 ms		8m	11m	14m	11m

of data is used. The phase from the second 20 ms data integration is subtracted by the phase from the first 20 ms data integration and divided by 20 ms. The result is shown in figure 4, which shows that the standard deviation is about 2 Hz at S/N of –34 dB. The biases in the measurement are very small about 0.1 Hz. The phase transition can also be determined by the phase measurements. A few early and late correlations around correlation peak are performed with a reference of which the frequency is 2Hz off and the code is 100ns off. The highest three points from the correlation results are used to fit a second order equation. The location of the peak of the equation is the estimate of the correlation peak. The results in table 3 are the root mean square error in meters between the estimated value and known value. The results suggest the 40ms data integration is adequate for pseudo-range estimate.

VI. Overall sensitivity analysis

From the acquisition, first phase determination and tracking one can find the least sensitivity process of the overall operation. It appears that all the methods can achieve S/N of about –34 dB. However, only the acquisition method uses the most favorable signal condition. All the other processes the most favorable signal condition does not apply, because the frequency can be calculated very accurately. The degradation from digitization is less than 1 dB. Therefore, in the overall processing, the acquisition still limits of the sensitivity of the receiver, which is the expected result. The sensitivity of the receiver can achieve from –31 to –34 dB. This corresponds to an improvement of 12 to 15 dB.

VII. User position calculation

Once the tracking is accomplished the data can be decoded. From the data the positions of the satellites can be obtained. If the data is decoded correctly, the position of the satellite can be obtained very accurately. If the data is incorrectly decoded due to noise, catastrophic error may occur. However, this condition will not be considered. If the satellite positions are calculated correctly, the only error in calculating the user position is due to the inaccuracy of the pseudo-ranges. The pseudo-range is the distance from the user to a satellite, which is obtained from the peak of the C/A correlation. The peak error listed in Table 3 reflects the user position error. The user position error should have the same order of magnitude. If only four satellites are used to calculate user position, the error can be several times the error in the pseudo-range.

VIII. Summary

A software GPS receiver designed to process weak signals seems feasible. The sensitivity performance can be improved about 12 dB. The noise will affect the user location accuracy. It versus S/N will be provided.

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